Reg. No.: | | | | | | | | |

G. VENKATASWAMY NAIDU COLLEGE (AUTONOMOUS), KOVILPATTI – 628 502.

PG DEGREE END SEMESTER EXAMINATIONS - APRIL 2025.

(For those admitted in June 2023 and later)

PROGRAMME AND BRANCH: M.Sc., MATHEMATICS

SEM	CATEGORY	COMPONENT	COURSE CODE	COURSE TITLE
I	PART - III	CORE - 2	P23MA102	REAL ANALYSIS - I

Date & Session : 24.04.2025/AN **Time : 3 hours** **Maximum: 75 Marks**

Course Outcome	Bloom's K-level	Q. No.	**SECTION – A (10 X 1 = 10 Marks)** Answer ALL Questions.
CO1	K1	1.	If f is monotonic on $[a,b]$, then the set of all discontinuous of f is ____ a) infinite interval b) countable c) finite interval d) none of the above
CO1	K2	2.	Which of the following is convergent but not absolutely convergent? a) $\sum_{n=1}^{\infty}\frac{1}{n}$ b) $\sum_{n=1}^{\infty}\left(\frac{-1}{n}\right)$ c) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$ d) none of these
CO2	K1	3.	If $\alpha \nearrow$ on $[a,b]$, then which of the following is correct one? a) $\bar{I}(f,\alpha) \le \underline{I}(f,\alpha)$ b) $\underline{I}(f,\alpha) \le \bar{I}(f,\alpha)$ c) $\bar{I}(f,\alpha) < \underline{I}(f,\alpha)$ d) $\underline{I}(f,\alpha) < \bar{I}(f,\alpha)$
CO2	K2	4.	Write down the value of $\int_a^a f\, d\alpha$? a) a b) 1 c) $2a$ d) 0
CO3	K1	5.	Which one of the following set has a measure 0? a) $\mathbb{Q}$ b) $[0,1]$ c) $\mathbb{R}$ d) $\mathbb{Q}^c$
CO3	K2	6.	A property is said to hold almost everywhere on a subset S of $\mathbb{R}^1$ if it holds everywhere on S except for _________. a) an interval in S b) all infinite sets in S c) a set of measure 0 d) a set of measure 1
CO4	K1	7.	What is the limit of $f(p,q) = \frac{pq}{p^2+q^2}$ as $p \to \infty, q \to \infty$. a) 0 b) p c) q d) not exists
CO4	K2	8.	The series $\sum_{n=1}^{\infty}\frac{z^n}{n}$ has the radius of convergence____. a) $r = 0$ b) $r = \infty$ c) $r = 1$ d) $r = 1/n$
CO5	K1	9.	$\lim_{n\to\infty} f_n{}'(x) =$ ____ where $f_n(x) = \frac{\sin nx}{\sqrt{n}}$ if $x \in R, n = 1,2,3, \ldots.$ a) 0 b) 1 c) $\cos nx$ d) does not exist
CO5	K2	10.	A sequence of functions $\{f_n\}$ is said to be boundedly convergent on T if $\{f_n\}$ is_________ on T. a) totally bounded b) pointwise convergent and uniformly bounded c) uniformly bounded d) uniformly convergent and totally bounded

Course Outcome	Bloom's K-level	Q. No.	**SECTION – B (5 X 5 = 25 Marks)** Answer ALL Questions choosing either (a) or (b)
CO1	K2	11a.	If f is continuous on $[a,b]$ and if f' exists and is bounded in the interior that is $\lvert f'(x)\rvert \le A$ for all x in (a,b), then illustrate that f is of bounded variation on $[a,b]$. **(OR)**
CO1	K2	11b.	Write down the statement and proof of the Dirichlet's test.

CO2	K2	12a.	If $f \in R(\alpha)$ and if $g \in R(\alpha)$ on $[a,b]$, show that $c_1 f + c_2 g \in R(\alpha)$ on$[a,b]$ and $\int_a^b (c_1 f + c_2 g)\, d\alpha = c_1 \int_a^b f\ d\alpha + c_2 \int_a^b g\ d\alpha$. **(OR)**
CO2	K2	12b.	Assume that $\alpha \nearrow$ on $[a,b]$. If $f \in R(\alpha)$, Show that $f^2 \in R(\alpha)$ on $[a,b]$.
CO3	K3	13a.	Write down the statement and the proof of the First mean-value theorem for Riemann-Stieltjes integrals. **(OR)**
CO3	K3	13b.	Write down the statement and the proof of second fundamental theorem of integral calculus.
CO4	K3	14a.	If a series is convergent with sum S, then illustrate that it is also $(C,1)$ summable with Cesaro sum S. **(OR)**
CO4	K3	14b.	Write down the statement of Abel's limit Theorem and illustrate it.
CO5	K4	15a.	Prove that $\lim_{n\to\infty} \int_0^1 f_n(x)dx \neq \int_0^1 \lim_{n\to\infty} f_n(x)dx$ where $f_n(x) = n^2 x(1-x)^n$ where $x \in R, n = 1,2,3,\dots$ **(OR)**
CO5	K4	15b.	Write down the statement of Dirichlet's test for uniform convergence and illustrate it.

Course Outcome	Bloom's K-level	Q. No	**<u>SECTION – C</u> (5 X 8 = 40 Marks)** **Answer <u>ALL</u> Questions choosing either (a) or (b)**
CO1	K4	16a.	Let f be of bounded variation on [a,b] and assume that $c \in (a,b)$.Show that f is of bounded variation on [a,c] and [c,b] and $V_f(a,b) = V_f(a,c) + V_f(c,b)$. **(OR)**
CO1	K4	16b.	Let $\sum a_n$ be an absolutely convergent series having sum s. Then show that every arrangement of $\sum a_n$ also converges absolutely and has sum s.
CO2	K5	17a.	State and prove the integration by parts formula. **(OR)**
CO2	K5	17b.	Assume $f \in R(\alpha)$ on $[a,b]$ and assume that α has continuous derivative α' on $[a,b]$. Prove that the Riemann integral $\int_a^b f(x)\alpha'(x)dx$ exists and $\int_a^b f(x)d\alpha(x) = \int_a^b f(x)\alpha'(x)dx$.
CO3	K5	18a.	Assume that α is of bounded variation on $[a,b]$. Let $V(x)$ denote the total variation of α on $[a,x]$ if $a < x \le b$ and let $V(a) = 0$. Let f defined and bounded on $[a,b]$. If $f \in R(\alpha)$ on $[a,b]$, then prove that $f \in R(V)$ on $[a,b]$. **(OR)**
CO3	K5	18b.	State and prove the Lebesgue's criterion for Riemann-integrability.
CO4	K5	19a.	State and prove the Merten's Theorem. **(OR)**
CO4	K5	19b.	State and prove the Bernstein's Theorem.
CO5	K6	20a.	Write down the statement of Dirichlet's test for uniform convergence and compose the proof of it. **(OR)**
CO5	K6	20b.	Let $\{f_n\}$ be a boundedly convergent sequence on [a,b]. Assume that each $f_n \in R$ on [a,b] and that the limit function $f \in R$ on [a,b]. Assume also that there is a partition P of [a,b], say $P = \{x_0, x_1, \dots, x_m\}$ such that on every subinterval [c,d] not containing any of the points x_k, the sequence $\{f_n\}$ converges uniformly to f. Then prove that $\lim_{n\to\infty} \int_a^b f_n(t)\,dt = \int_a^b \lim_{n\to\infty} f_n(t)\,dt = \int_a^b f(t)\,dt$